Bayes Theorem

In our mathematics, calculations are done based on the existing data, but probability is based on the future data/predictions.

e.g: Tossing a coin, we can find the probablility that there are 50% chances for head,50% chances for tail.

₼ P(A) = (no. of chances for an event)/(Total no of chances)

P(head in a coin)= 1/2

P(even no. in a die)=3/6

To understand Bayes Theorem it is essential to know

* Conditional Probability
* Independent Events

Conditional Probability:

Probability refers to how many percentage of chances that the event occurs, where as conditional probability is the percentage of chances with a given condition.

For ex: Lets take a layman context,

Event1 : route starts from Coimb to ooty

Event2 : route starts from coimb to bangalore.

The conditional probability is that the destination bangalore should be reached through ooty which indirectly means the first condition should have been achieved.

₼ P(B/A) = which event has occurred/Total events

P(B/A) = P(AΩB)/P(A)

Another Example could be

Event1 : A die has an even number

Event2: A die has a number less than 4

P(B/A) = P(BΩA)/P(A)

P(B/A) = (1/6)/(3/6)

Independent Events:

When one event does not depend on another event, it is known as

Independent Event.

E.g: Tossing two coins in two hands.

The probability of getting same head in both the hands.

P(AΩB) = P(A)\*P(B)

If we apply conditional probability to the Independent event,

P(AΩB) = P(A/B)\*P(B)

₼ P(A)\*P(B) = P(A/B)\*P(B)

P(A) = P(A/B)

Which says that the conditional probability of an independent probability is equal to the probability.

BAYES THEOREM

Let us say we A,B,C are independent events. For our understanding lets project them as

A 🡪 Applying for a Medical College

B 🡪 Applying for an engineering College

C 🡪 Applying for an Arts College

D 🡪 some other event related to previous event.

P(D) – The probability of getting rejected by any of the colleges

P(D) = P(A) P(D/A) + P(B) P(D/B)+ P(C)P(D/C)

P(A/D) –The Probability of rejection from a medical college

P(A/D) = P(AΩD)/P(D)

P(A/D) = p(AΩD)/P(A)P(D/A)+P(B)P(D/B)+P(C)P(D/C)

P(B/D) – The Probability of rejection from an engineering college

P(B/D) = P(BΩD)/P(D)

P(B/D) = P(BΩD)/P(A)P(D/A)+P(B)P(D/B)+P(C)P(D/C)

P(C/D) – The probability of rejection from an Arts College

P(C/D) = P(CΩD)/P(D)

P(C/D) = P(CΩD)/P(A)P(D/A)+P(B)P(D/B)+P(C)P(D/C)

This is your Bayes Theorem(P(A/D),P(A/B),P(A/C)

PROBLEM

A factory has 2 machines I and II.Machine I and II produce 30% and 70% of items respectively.Further 3% of items produced by machine I are defective and 4% of items produced by machine II are defective.

An item is drawn at random.If the drawn item is defective,find the probability it was produced by machine II.

Solution: Let A1 and A2 be the events produced by the machine I and II respectively.

P(A1) = 0.3

P(A2) = 0.7

Let B be the event of drawing a defective item.

P(B/A1) = 3/100 = .03

P(B/A2) = 4/100 = .04

What is asked? P(A2/B)

P(A2/B) = P(A2ΩB)/P(B)

P(B)=P(A1)P(B/A1)+P(A2)P(B/A2)

P(A2/B) = P(A2ΩB)/P(A1)P(B/A1)+P(A2)P(B/A2)

P(A2/B) = P(A2)\*P(B/A2)/P(A1) P(B/A1)+P(A2)P(B/A2)

=( 0.7\*0.04)/(0.3\*0.03)+(0.7\*0.04)

=0.028/0.009+0.028

=0.028/0.037

=0.76

The probability that it was produced from machine II is 76%

This is the sole application of Bayes theorem.